A new approach for the model analysis of nonlinear multistory structures

Roberto Villaverde
University of California, Irvine, USA

ABSTRACT: An approximate procedure is introduced to analyze nonlinear multistory structures within the framework of the conventional response spectrum method. Its derivation is based on the use of nonlinear response spectra and an approximate decomposition of the equation of motion for multidegree-of-freedom nonlinear systems. The decomposition is attained by considering the nonlinear terms in this equation of motion as additional external forces and, thus, by interpreting it as the equation of motion of linear systems with the initial properties of the nonlinear ones but subjected to a modified set of inertia forces. For simplicity, the procedure is herein limited to elastoplastic systems of the shear-beam type. Its accuracy is evaluated by comparing the approximate and step-by-step integration solutions of systems with three and ten degrees of freedom when subjected to three different earthquake ground motions.

1 INTRODUCTION

It is well known that under the current philosophy of earthquake-resistant design, a structure is supposed to resist strong and severe ground motions through inelastic deformation. As a result, the capability of a structure to withstand a strong earthquake is measured by its ability to resist the inelastic deformations or ductility demands imposed by such an earthquake. Thus, to assess such inelastic deformations one needs to carry out analyses in which the nonlinear characteristics of the structure are taken into account. Presently, however, a reliable nonlinear analysis of a multidegree-of-freedom (MDOF) system may be only possible through a step-by-step integration of its equation of motion, which is a procedure that is costly and cumbersome in most cases and certainly unacceptable for preliminary designs. Furthermore, such a numerical procedure does not facilitate a direct visualization of structural behavior, or the sensitivity of the response of a structure to design changes and variations in the characteristics of the ground motion that excites it.

In the search for simpler methods of analysis and attracted by the simplicity and rationality of the response spectrum technique that has been widely used in the analysis of linear structures, several investigators have thus proposed different procedures to extend the use of this technique to nonlinear systems (Newmark, 1970, Shibata & Sozen 1976). In general, the trend has been the use of inelastic response spectra in combination with an elastic modal analysis (Newmark 1970, Tansirikongkol & Pecknold 1978), or the use of linear response spectra in conjunction with equivalent linearization techniques (Caughey 1963, Iwan 1968, Tansirikongkol & Pecknold 1978). It seems, however, that such procedures are too complicated or too empirical and, thus, judging from their relatively low use, still unsatisfactory for general applications (Clough et al. 1965, Veletsos & Vann 1971, Anagnostopoulos et al. 1978, Hadjian 1982).

It is the purpose of this paper to propose an alternative method by means of which a nonlinear MDOF structure may be analized by an extension of the conventional response spectrum technique. It is intended to serve primarily as a tool in preliminary designs, where a rigorous step-by-step integration approach may be prohibitively costly and time consuming. As such, its simplicity is empasized over its accuracy. Also, for the sake of introducing the basic ideas without the complications of the general case, it is derived specifically for elastoplastic shear systems with linear damping forces at all response levels.

2 APPROACH

Using the definition of a response spectrum, it is easy to understand that it is also possible to construct response spectra for nonlinear systems. That is, if a certain load-deformation type of behavior is established, by a step-by-step integration one can obtain for a given ground disturbance plots that relate the maximum response of a series of simple oscillators to their natural frequencies and damping ratios. Examples of these are the inelastic response spectra obtained by Riddell and Newmark (1979) for systems with elastoplastic, bilinear, and bilinear with stiffness degradation load-deformation curves. Figure 1 shows a sample of such nonlinear response spectra, where the parameter used to describe the nonlinearity of the oscillators is their yield deformation.

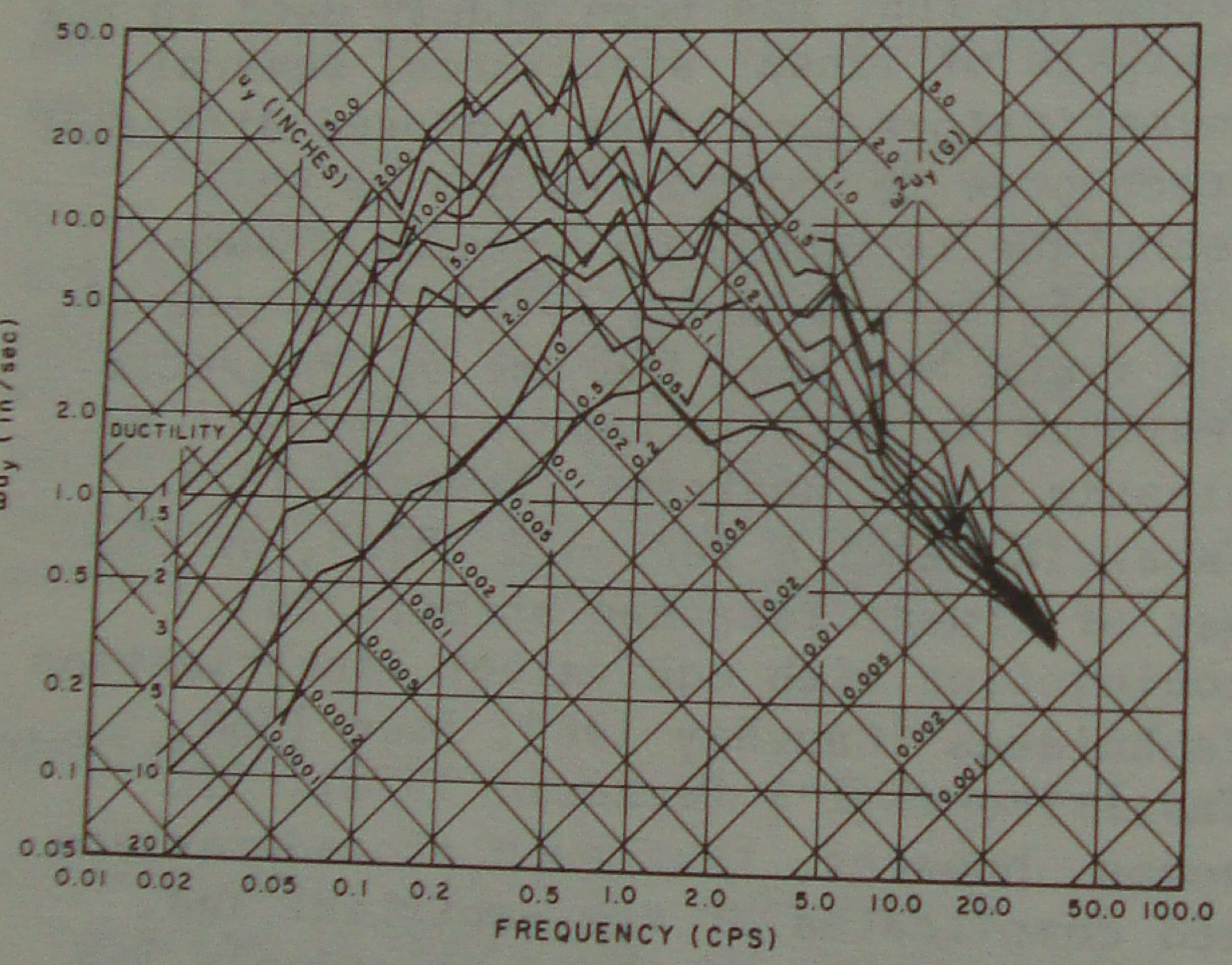

Figure 1. Nonlinear response spectra for elastoplastic systems with 2% damping; E-W component of El Centro, 1940, earthquake

The concept of nonlinear spectra shows thus that one can determine the maximum response of a nonlinear single-degree-of-freedom (SDOF) system using information based on its initial elastic properties (natural frequency and damping ratio), together with some information about its nonlinear behavior (yield deformation, in the case of elastoplastic systems). This fact further suggests that it may be equally possible to estimate the maximum response of a nonlinear MDOF system from a nonlinear response spectrum if one knows its initial elastic properties and the load-deformation characteristics of each of its floors. In addition, if one takes into account that such initial properties can be adequately described by the system's mode shapes, natural frequencies, and damping ratios, it seems feasible that the sougth simplified method can be based on a modal decomposition in combination with the use of nonlinear response spectra.

The method herein proposed is therefore developed seeking this approach. For this purpose, in what follows an equation is derived first to express the maximum response of a nonlinear SDOF system in terms of its natural frequency, damping ratio, and yield displacement. Thereafter, the equation of motion representing the behavior of a nonlinear MDOF system is manipulated to transform it into an approximate series of simple equations, where each represents the equation of motion of a nonlinear SDOF system. Lastly, the solution to each of these equations is related to the excitation nonlinear response spectrum by comparing it with the expression for the maximum response of a nonlinear SDOF system and by introducing a defintion of a modal yield displacement.

3 MAXIMUM RESPONSE OF NONLINEAR SDOF SYSTEM

Consider the SDOF system depicted in Figure 2(a), which is defined by its mass M, damping constant C, initial stiffness K, yield displacement x_y, and the load-deformation relationship shown in Figure 2(b). The equation of motion for this system when subjected to a ground acceleration $\ddot{U}_g(t)$ may be written as

$$M\ddot{x}(t) + C\dot{x}(t) + F_R(x) = -M\ddot{U}_g(t) \qquad (1)$$

where x denotes the displacement of the mass M relative to the ground, and $F_R(x)$, a function of x that needs to be read from Figure 2(b), represents the restoring force in the spring of the system. However, following the idea suggested by Geschwindner (1981) and Dungar (1982), for every point in the load-deformation curve the restoring force may be written in terms of an elastic and a corrective component as (see Figure 3)

$$F_R(x) = K\,x(t) - K[X(t) - X_y(t)] \qquad (2)$$

where X and X_y are stepwise continuous functions that vary from branch to branch as indicated in Table 1. Note that $X_y(t)$ varies randomly between upper and lower bounds, as indicated in Figure 4. Upon substitution of equation 2 into equation 1, rearranging terms, and dividing through by M, one then obtains

$$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \omega^2 x(t) = -\{\ddot{U}_g(t) - \omega^2[X(t) - X_y(t)]\} \quad (3)$$

where $\omega = \sqrt{K/M}$ is the natural frequency of the system and $\xi = C/2\omega m$ its damping ratio, both at low response levels. However, if the right-hand side of equation 3 is interpreted as an effective ground acceleration defined as

$$\ddot{U}_{eff}(t) = \ddot{U}_g(t) - \omega^2[X(t) - X_y(t)] \quad (4)$$

the equation of motion of the system can be alternatively expressed as

$$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \omega^2 x(t) = -\ddot{U}_{eff}(t) \quad (5)$$

Thus, the vibrational motion of a nonlinear SDOF system can be viewed as that of a linear one with the initial properties of the former, but subjected to an effective ground acceleration as given by equation 4. As a consequence, the methods for linear systems can be used to find its maximum response, provided the actual ground acceleration is substituted by the effective one.

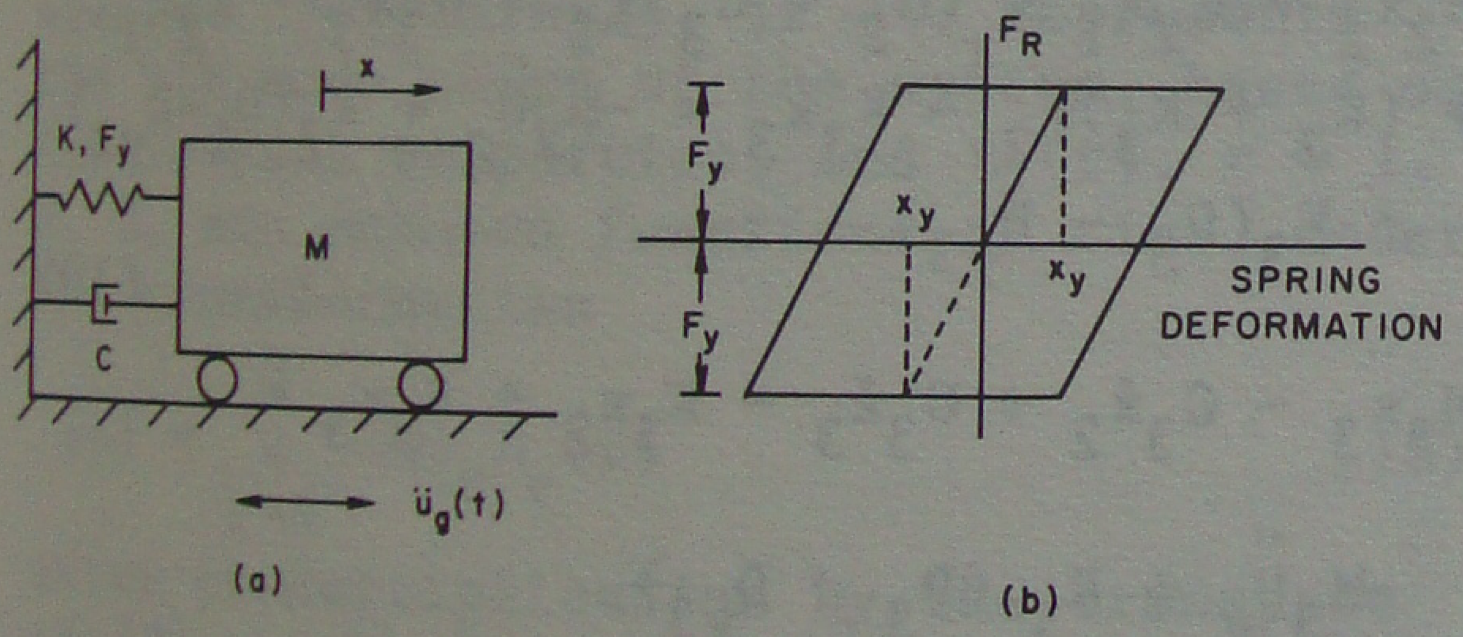

Figure 2. Elastoplastic single-degree-of freedom system

Accordingly, in terms of Duhamel integral the solution of equation 3 results as

$$x(t) = -(1/\omega)\int_0^t e^{-\xi\omega(t-\tau)}\{\ddot{U}_g(\tau) - \omega^2[X(\tau) - X_y(\tau)]\}\sin\omega'(t-\tau)d\tau \quad (6)$$

where $\omega' = \omega\sqrt{1-\xi^2}$. The maximum of this solution, however, represents the ordinate in the nonlinear response spectrum of $\ddot{U}_g(t)$ corresponding to a natural frequency ω, a damping ratio ξ, and a yield displacement x_y; thus, if $SD(\omega, \xi, x_y)$ denotes such an ordinate, one may write

$$SD(\omega, \xi, x_y) = \left|(1/\omega)\int_0^t e^{-\xi\omega(t-\tau)}\{\ddot{U}_g(\tau) - \omega^2[X(\tau) - X_y(\tau)]\}\sin\omega'(t-\tau)d\tau\right|_{max} \quad (7)$$

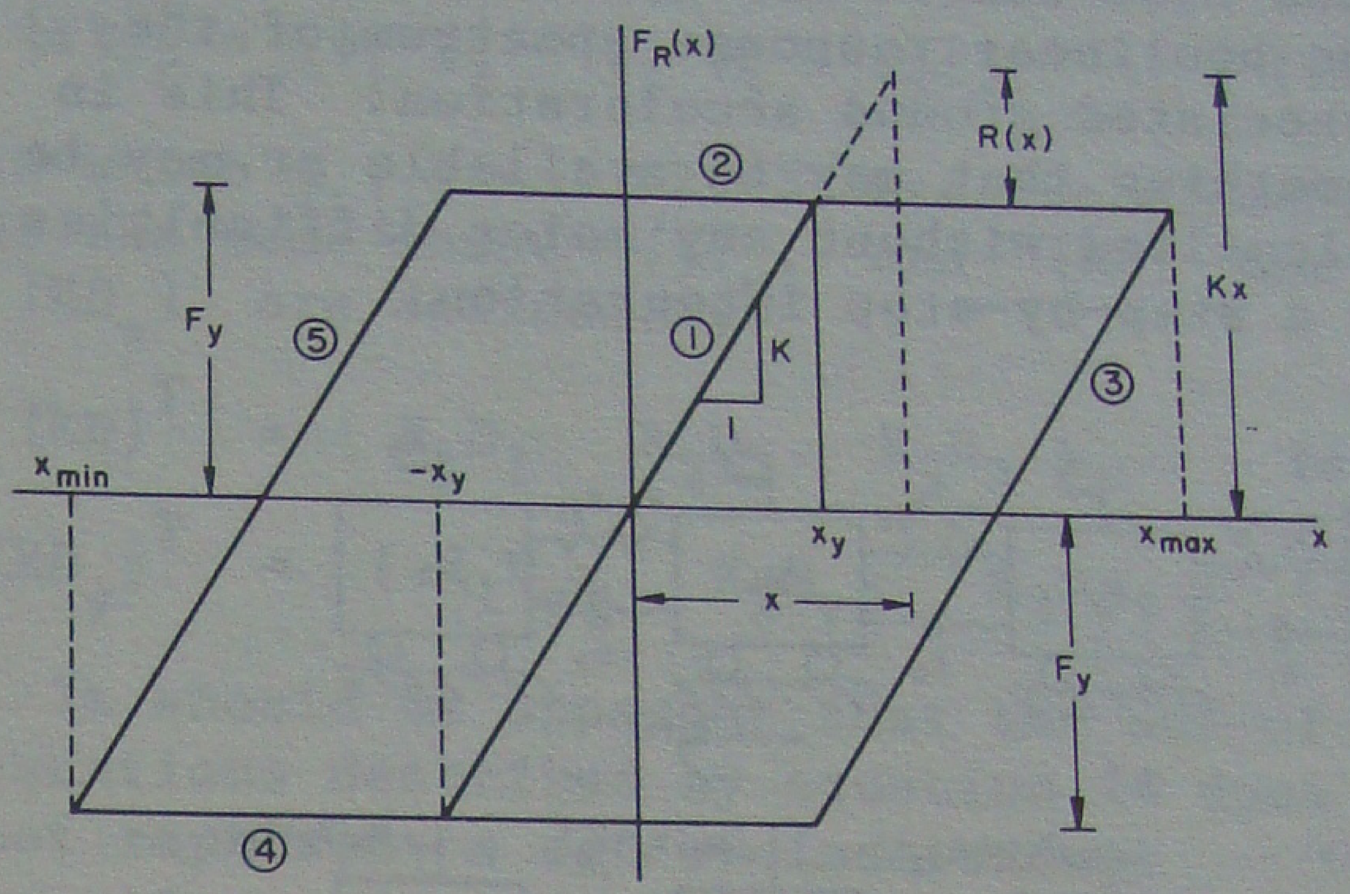

Figure 3. Load-deformation curve

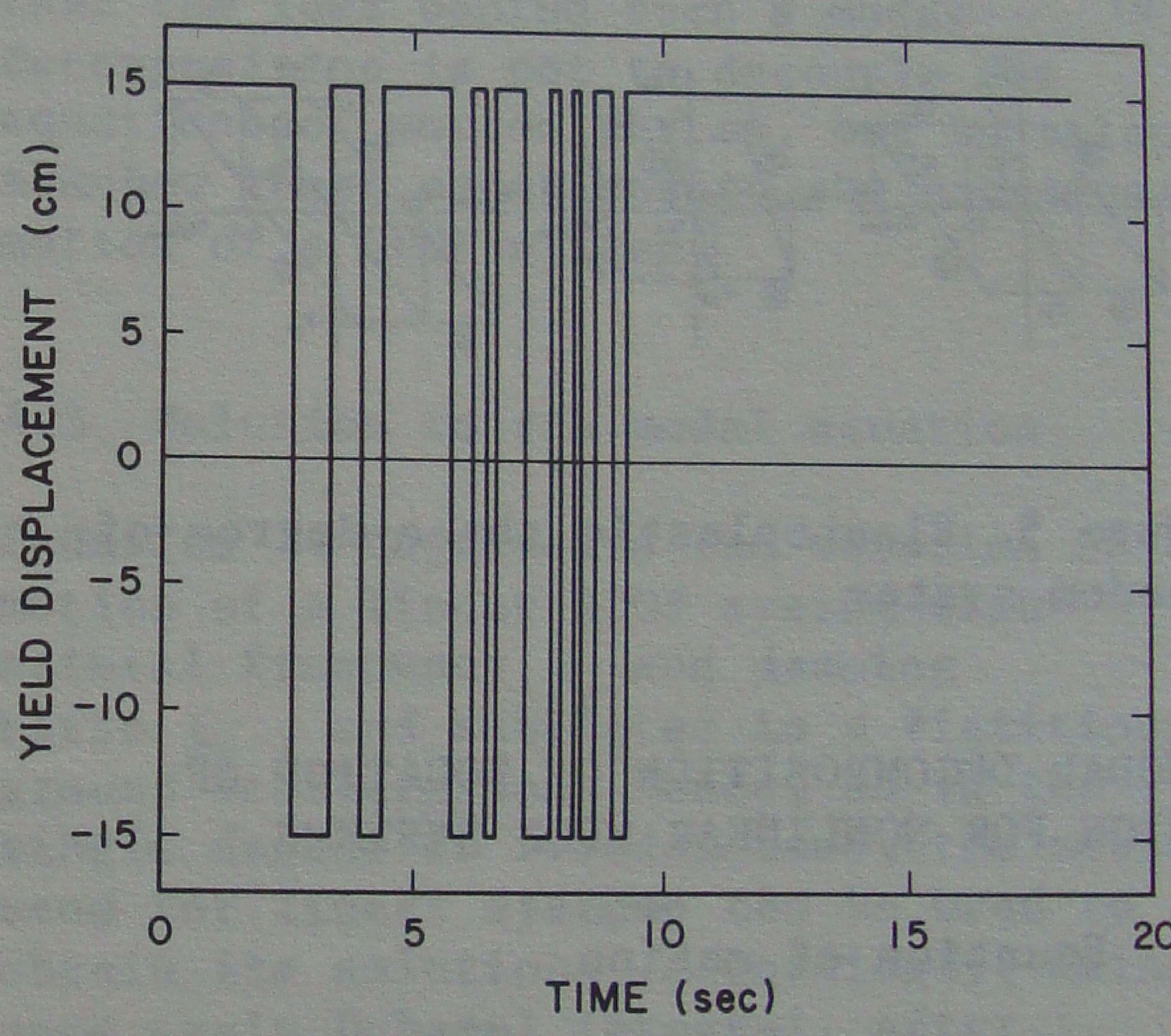

Figure 4. Variation of X_y with time

Table 1. Values of X and X_y

Branch	X	X_y
1	x_y	x_y
2	x	x_y
3	x_{max}	x_y
4	x	$-x_y$
5	x_{min}	$-x_y$

This expression may be considered an analytic representation of a nonlinear response spectrum, which will be helpful later on in the treatment of systems with several degrees of freedom. It should be noted that in this equation the variation of $X(\tau)$ and $X_y(\tau)$ is not known in advance, and therefore for a given excitation $U_g(t)$ one cannot use it to calculate response spectrum ordinates. Its usefulness, rather, lies on the fact that an integral of the form of its right-hand side can be easily determined from the nonlinear response spectrum of the associated ground acceleration. This is something that may be available or may be calcualted without any major difficulties by a step-by-step integration.

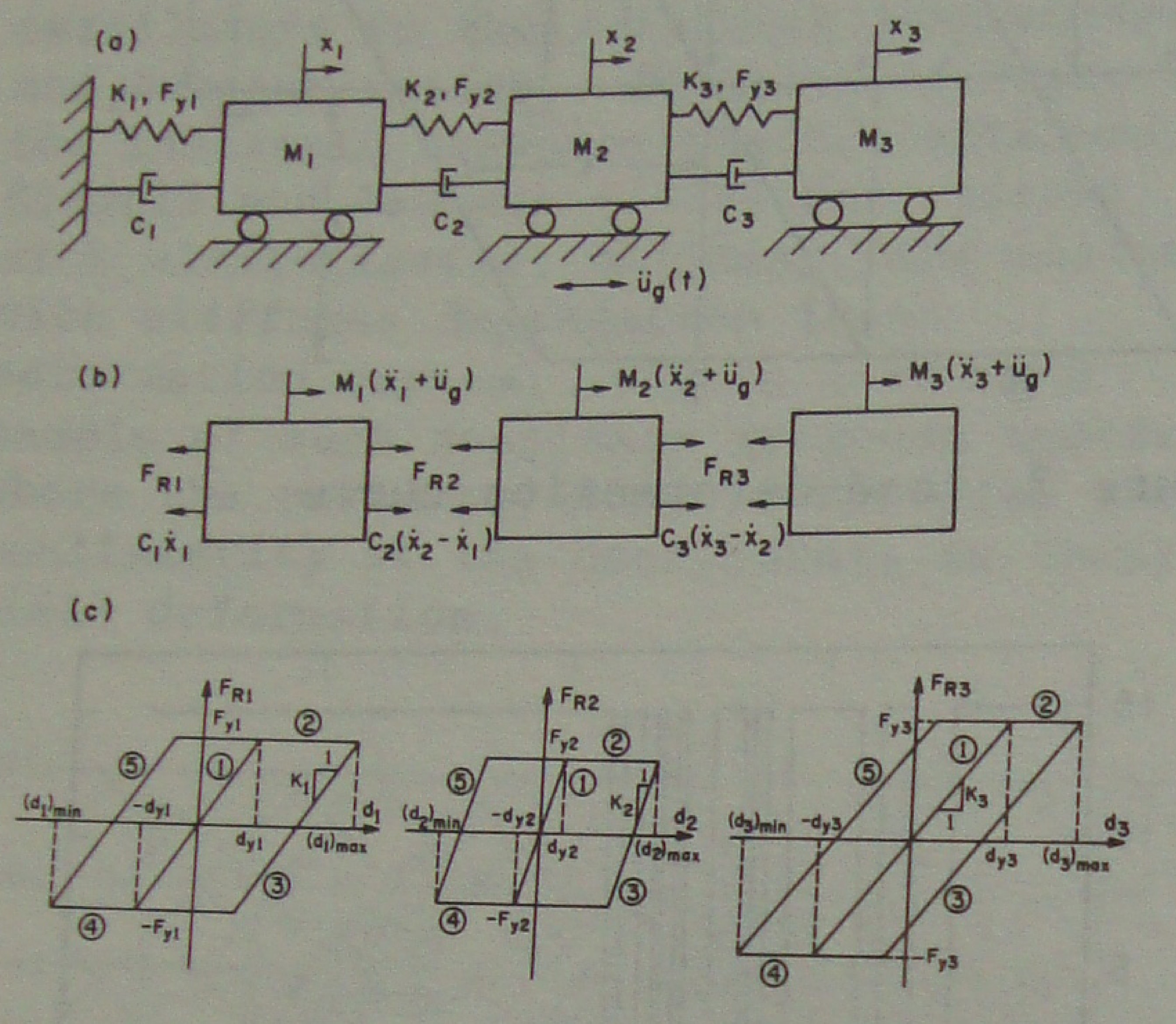

Figure 5. Elastoplastic three-degree-of-freedom system

4 MODAL DECOMPOSITION OF EQUATION OF MOTION FOR NONLINEAR MDOF SYSTEMS

4.1 Equation of motion

If elastoplastic behavior is assumed for all the floors of an MDOF system, but each characterized by its own initial stiffness and yield displacement, it is evident from the above discussion that its vibrational motion may also be represented by the equation of motion of an elastic system subjected to an effective ground acceleration. To derive such an equation, consider then, without any loss of generality, the three-degree-of-freedom system shown in Figure 5(a), whose springs have the load-deformation characteristics presented in Figure 5(c). According to the free-body diagrams shown in Figure 5(b), the equations of motion for each of the masses of the system are

$$M_1(\ddot{x}_1 + \ddot{U}_g) + C_1\dot{x}_1 + F_{R1} - C_2(\dot{x}_2 - \dot{x}_1) - F_{R2} = 0$$
$$M_2(\ddot{x}_2 + \ddot{U}_g) + C_2(\dot{x}_2 - \dot{x}_1) + F_{R2} - C_3(\dot{x}_3 - \dot{x}_2) - F_{R3} = 0 \quad (8)$$
$$M_3(\ddot{x}_3 + \ddot{U}_g) + C_3(\dot{x}_3 - \dot{x}_2) + F_{R3} = 0$$

where M_1, M_2, M_3 are such masses; C_1, C_2, C_3 its damping constants; F_{R1}, F_{R2}, F_{R3} the restoring forces in its springs, as given by Figure 5(c); and x_1, x_2, x_3 the relative displacements of the masses M_1, M_2, M_3, respectively. However, if the same procedure employed for the SDOF system is now used to express the restoring forces F_{R1}, F_{R2}, and F_{R3} in terms of elastic and corrective components, these restoring forces can be written as

$$F_{Ri} = K_i d_i - K_i D_i + K_i D_{yi}, \quad i=1,2,3 \quad (9)$$

where d_1, d_2, d_3 denote deformations in the springs of the system, and D_1, D_2, D_3, and D_{y1}, D_{y2}, D_{y3} are stepwise continuous functions that vary from branch to branch as indicated in Table 2. If it is considered that $d_1 = x_1$, $d_2 = x_2 - x_1$ and $d_3 = x_3 - x_2$, the equations of motion of the system may be therefore expressed as

$$M_1\ddot{x}_1 + (C_1 + C_2)\dot{x}_1 - C_2\dot{x}_2 + (K_1 + K_2)x_1 - K_2x_2 = -M_1\ddot{U}_g + K_1(D_1 - D_{y1}) - K_2(D_2 - D_{y2})$$

$$M_2\ddot{x}_2 - C_2\dot{x}_1 + (C_2 + C_3)\dot{x}_2 - C_3\dot{x}_3 - K_2x_1 + (K_2 + K_3)x_2 - K_3x_3 = -M_2\ddot{U}_g + K_2(D_2 - D_{y2}) - K_3(D_3 - D_{y3}) \quad (10)$$

$$M_3\ddot{x}_3 - C_3\dot{x}_2 + C_3\dot{x}_3 - K_3x_2 + K_3x_3 = -M_3\ddot{U}_g + K_3(D_3 - D_{y3})$$

which in matrix form can be put into the form

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F_I\}_{eff} \quad (11)$$

where $[M]$, $[C]$, $[K]$ are respectively the system's initial mass, damping, and stiffness matrices, $\{x\}$ its vector of relative displacements, and $\{F_I\}_{eff}$ is a vector that can be interpreted as a vector of effective inertia forces defined by

$$\{F_I\}_{eff} = -[M]\{J\}\ddot{U}_g + \{K(D-D_y)\} \quad (12)$$

in which $\{K(D-D_y)\}$ is a vector whose entries are of the form $K_i(D_i - D_{yi}) - K_{i+1}(D_{i+1} - D_{y(i+1)})$ and $\{J\}$ is a unit vector.

Table 2. Values of D_i and D_{yi}

Branch	D_1	D_2	D_3
1	d_{y1}	d_{y2}	d_{y3}
2	d_1	d_2	d_3
3	$(d_1)_{max}$	$(d_2)_{max}$	$(d_3)_{max}$
4	d_1	d_2	d_3
5	$(d_1)_{min}$	$(d_2)_{min}$	$(d_3)_{min}$
Branch	**D_{y1}**	**D_{y2}**	**D_{y3}**
1	d_{y1}	d_{y2}	d_{y3}
2	d_{y1}	d_{y2}	d_{y3}
3	d_{y1}	d_{y2}	d_{y3}
4	$-d_{y1}$	$-d_{y2}$	$-d_{y3}$
5	$-d_{y1}$	$-d_{y2}$	$-d_{y3}$

4.2. Equation of motion in terms of normal coordinates

The inspection of equation 11 indicates thus that, as in the case of a single degree of freedom, the equation of motion of a nonlinear MDOF system can be interpreted as the equation of motion of an associated linear system subjected to effective inertia forces. As such, equation 11 can be the subject of a modal decomposition as it would be the case with the equation of motion of a true linear system. Accordingly, if $\{\psi\}_r$ represents the rth mode shape of the system, based on its initial properties, and $\eta_r(t)$, r = 1, 2, 3, are unknown functions of time, under the transformation

$$\{x\} = \sum_{r=1}^{3} \{\psi\}_r \eta_r(t) \tag{13}$$

after premultiplication by the transpose of $\{\psi\}_r$, and after making use of the orthogonality properties of mode shapes, equation 11 may be simplified to the following series of equations:

$$\ddot{\eta}_r + 2\,\xi_r \omega_r \dot{\eta}_r + \omega_r^2\, \eta_r = (\ddot{U}_r)_{eff}, \quad r=1,2,3 \tag{14}$$

where ξ_r and ω_r signify respectively the rth damping ratio and natural frequency of the system; and $(\ddot{U}_r)_{eff}$ represents an effective ground acceleration in the rth mode defined by

$$(\ddot{U}_r)_{eff} = \{\psi\}_r^T \{F_I\}_{eff} \,/\, \hat{M}_r \tag{15}$$

which in the light of equation 12 may also be written as

$$(\ddot{U}_r)_{eff} = -\,\alpha_r \ddot{U}_g + \{d\psi\}_r^T [\{KD\} - \{KD_y\}] / \hat{M}_r \tag{16}$$

Here α_r and $\hat{M}_r$ are respectively the rth participation factor and rth generalized mass of the system, $\{d\psi\}_r^T$ is a vector of the form

$$\{d\psi\}_r^T = \{\psi_1 \;\; \psi_2 - \psi_1 \;\; \psi_3 - \psi_2\}_r \tag{17}$$

in which ψ_i, i = 1, 2, 3, are the elements of the mode shape $\{\psi\}_r$, and $\{KD\}$ and $\{KD_y\}$ are defined by

$$\{KD\}^T = \{\, K_1D_1 \quad K_2D_2 \quad K_3D_3 \,\} \tag{18}$$

$$\{KD_y\}^T = \{\, K_1D_{y1} \quad K_2D_{y2} \quad K_3D_{y3} \,\} \tag{19}$$

It should be apparent that the set of equations described by equation 14 does not represent a set of independent equations since $(\ddot{U}_r)_{eff}$ depends on the deformations d_1, d_2, and d_3, which in turn depend on η_1, η_2 and η_3. Note, however, that the idea behind such a modal decomposition is not to decouple the equations of motion per se, but to relate somehow these equations to the equation of motion of a SDOF system.

4.3 Solution to rth modal equation

Equation 14 represents the equation of motion of a linear SDOF system with natural frequency ω_r and damping ratio ξ_r, and subjected to a fictitious ground acceleration: hence, as in the single degree of freedom case, the methods used for linear systems can be used to obtain its solution. Accordingly, using once again Duhamel integral, after substitution of equation 16 the solution of equation 14 results as

$$\eta_r(t) = -\,(1/\omega_r)\int_0^t e^{-\xi_r\omega_r(t-\tau)}[\alpha_r\ddot{U}_g - \{d\psi\}_r^T(\{KD\} - \{KD_y\}) / \hat{M}_r] \sin \omega_r'(t-\tau)d\tau \tag{20}$$

which can also be written as

$$\eta_r(t) = -\,(\alpha_r/\omega_r)\int_0^t e^{-\xi_r\omega_r(t-\tau)}[\ddot{U}_g - \omega_r^2\,(X_r - X_{yr})] \sin \omega_r'(t-\tau)d\tau \tag{21}$$

if

$$X_r - X_{yr} = \{d\psi\}_r^T(\{KD\} - \{KD_y\})/(\alpha_r \omega_r^2 \hat{M}_r) \quad (22)$$

By comparing equation 21 with equation 6, it may be then seen that the former may be interpreted as the equation of a nonlinear SDOF system with initial frequency ω_r, damping ratio ξ_r, and a yield displacement that can be calculated from equation 22. As such, the maximum value of $\eta_r(t)$ may be obtained from the nonlinear response spectrum of U_g, and thus one can write

$$|\eta_r|_{max} = \alpha_r SD(\omega_r, \xi_r, x_{yr}) \quad (23)$$

where x_{yr} represents a modal yield displacement that still remains to be determined.

In the light of equations 13 and 23, it is therefore evident that the maximum response of a nonlinear MDOF system can also be estimated by means of a combination of response spectrum values, which, in similarity with the linear case, and if N represents the system's total number of degrees of freedom, may be approximated as

$$\{d\}_{max} = \sqrt{\sum_{i=1}^{N} [\alpha_r \{d\psi\}_r SD(\omega_r, \xi_r, x_{yr})]^2} \quad (24)$$

when maximum distortions are the response of interest, and similar expressions for other types of response. Observe that since the response of a nonlinear system may be interpreted as the response of a linear one subjected to an effective ground acceleration, it has been implicitly assumed in the establishment of the above equation that the approximate rules that are normally used to combine the modal responses of linear systems are also applicable to nonlinear ones.

4.4 Modal yield displacements

As mentioned earlier, the use of equation 24 requires the definition of a modal yield displacement whereby one can enter a nonlinear response spectrum and determine spectral values for each of the modes of the system. To arrive to this definition, one may then consider that equation 22 is satisfied when

$$X_r = \{d\psi\}_r^T\{KD\}/(\alpha_r \omega_r^2 \hat{M}_r) \quad (25)$$

$$X_{yr} = \{d\psi\}_r^T\{KD_y\}/(\alpha_r \omega_r^2 \hat{M}_r) \quad (26)$$

and that X_r and X_{yr} should behave like the functions X and X_y that describe the behavior of a SDOF system, as shown in Table 1. However, if this last equation is rewritten as

$$X_{yr} = (\sum_{i=1}^{N} d\psi_i(r) \quad K_i \; D_{yi})/(\alpha_r \omega_r^2 \hat{M}_r) \quad (27)$$

where $d\psi_i(r)$ represents the ith element of $\{d\psi\}_r$, one may notice that X_{yr} is not a quantity that varies between the negative and a positive of a constant value. It varies, rather, randomly in sign and in value as shown in Figure 6 since: (a) D_{yi}, a function of time, fluctuates between $-d_{yi}$ and $+ d_{yi}$ as the distortion of the ith floor of the system moves along its hysterisis loop; (b) $d\psi_i(r)$ is a quantity that may take positive and negative values; and (c) in general, not all the floors follow the same pattern of deformation, which means that not all of them are at the same branch of their corresponding load-deformation curves at the same time. Strictly speaking, therefore, X_{yr} does not represent the behavior of a nonlinear SDOF system, and hence an approximation of some sort is necessary for an adequate definition of a modal yield displacement.

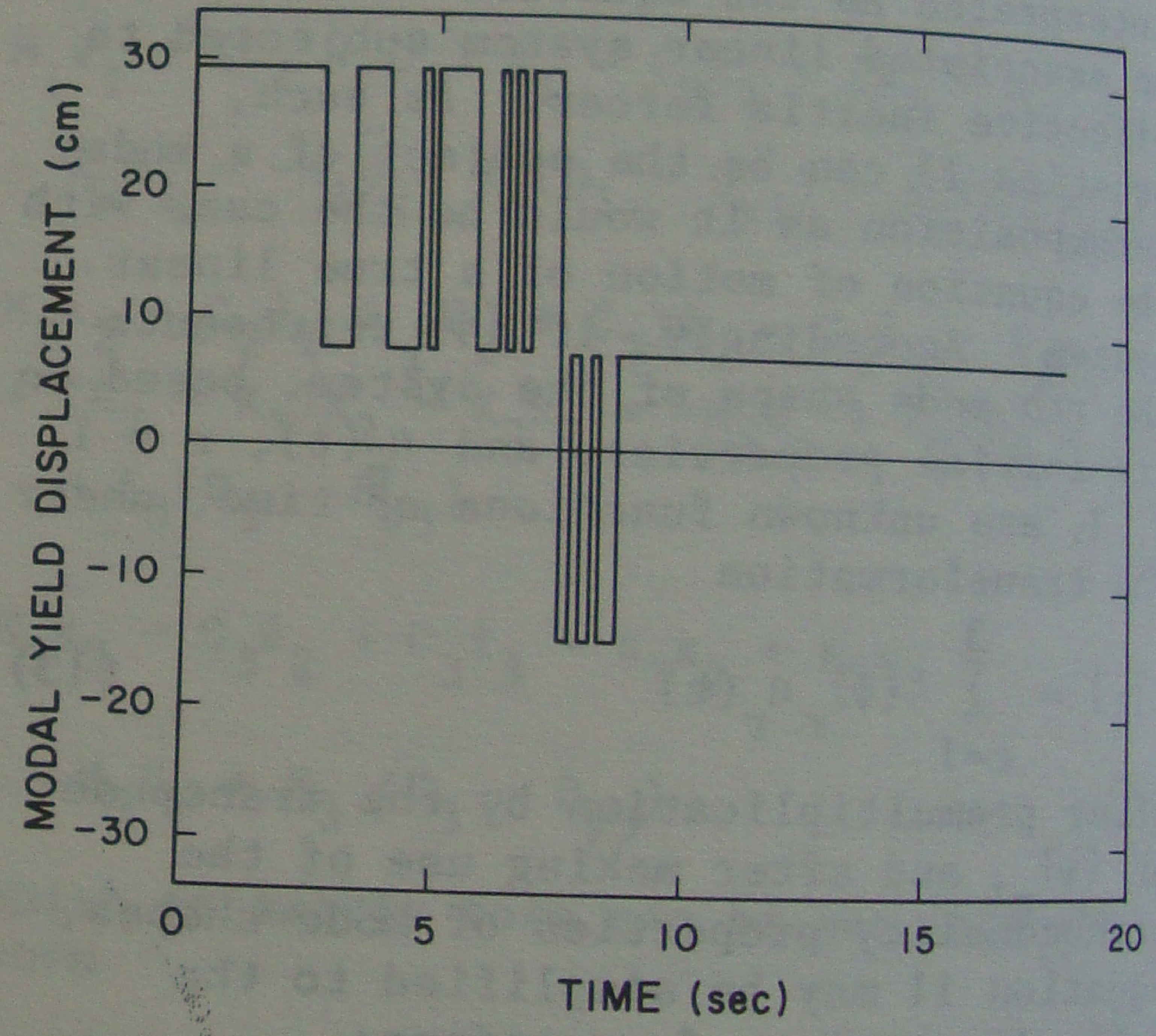

Figure 6. Variation of X_{yr} with time

A possible approximation for a modal yield dispalcement may be attained by observing that if X_{yr} is to characterize the behavior of a SDOF system, the following two conditions need to be satisfied: (1) $x_{yr}^2 = X_{yr}^2$ and (2) x_{yr}^2 = constant. From the first condition and in the light of equation 27, one can then write

$$x_{yr}^2 = [(\sum_{i=1}^{N} d\psi_i(r) \;\; K_i \; D_{yi})/(\alpha_r \omega_r^2 \hat{M}_r)]^2 \tag{28}$$

which can also be written as

$$x_{yr}^2 = \frac{1}{(\alpha_r \omega_r^2 \hat{M}_r)^2} [\sum_{i=1}^{N} (d\psi_i(r) K_i D_{yi})^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} (d\psi_i(r) K_i D_{yi})(d\psi_j(r) K_j D_{yj})] \tag{29}$$

However, if it is taken into consideration that, according to Table 2, $D_{yi} = \pm d_{yi}$, one can observe that the value of x_{yr} given by this expression is not a constant, but a function of time that varies according to the signs of the terms of the double summation. Hence, to have a modal yield displacement consistent with the concept of yield displacement of a SDOF system, as an approximation such double summation terms can be neglected to obtain the following definition

$$x_{yr} = \frac{1}{\alpha_r \omega_r^2 \hat{M}_r} \sqrt{\sum_{i=1}^{N} [d\psi_i(r) \;\; F_{yi}]^2} \tag{30}$$

where it has been taken into consideration that $D_{yi} = \pm d_{yi}$, and that $F_{yi} = K_i \; d_{yi}$, where F_{yi} represents the yield strength of the ith floor of the system.

Equation 30 should be considered thus the sought expression to calculate the modal yield displacements that are needed in the use of equation 24. Note that since the terms of the double summation in equation 29 have been neglected, a drawback of this approximation is that if a floor has an exceedingly large yield strength one obtains a large modal yield displacement for all modes and, hence, elastic behavior in all modes. Therefore, although in a realistic analysis all the floors of a system have yield strengths values that are finite and of the same order of magnitude, to avoid this kind of anomalous situation one may neglect in the computation of modal yield displacements the contribution of those floors that remain elastic at all times.

5 SUMMARY OF PROCEDURE

The calculation of the maximum displacement or distortion response of a nonlinear MDOF system by means of a response spectrum analysis consists thus of the following steps:

1. Calculation of the natural frequencies, mode shapes, participation factors, and damping ratios of the system on the basis of its initial elastic properties.
2. Calculation of a modal yield displacement for each of the modes of the system using the definition given by equation 30.
3. Determination of the spectral displacements that correspond to each of the natural frequencies, damping ratios, and modal yield displacements of the system from a specified nonlinear response spectrum.
4. Computation of vectors of maximum modal displacements or distortions and combination of these modal maxima using the square root of the sum of the squares rule, as indicated by equation 24.

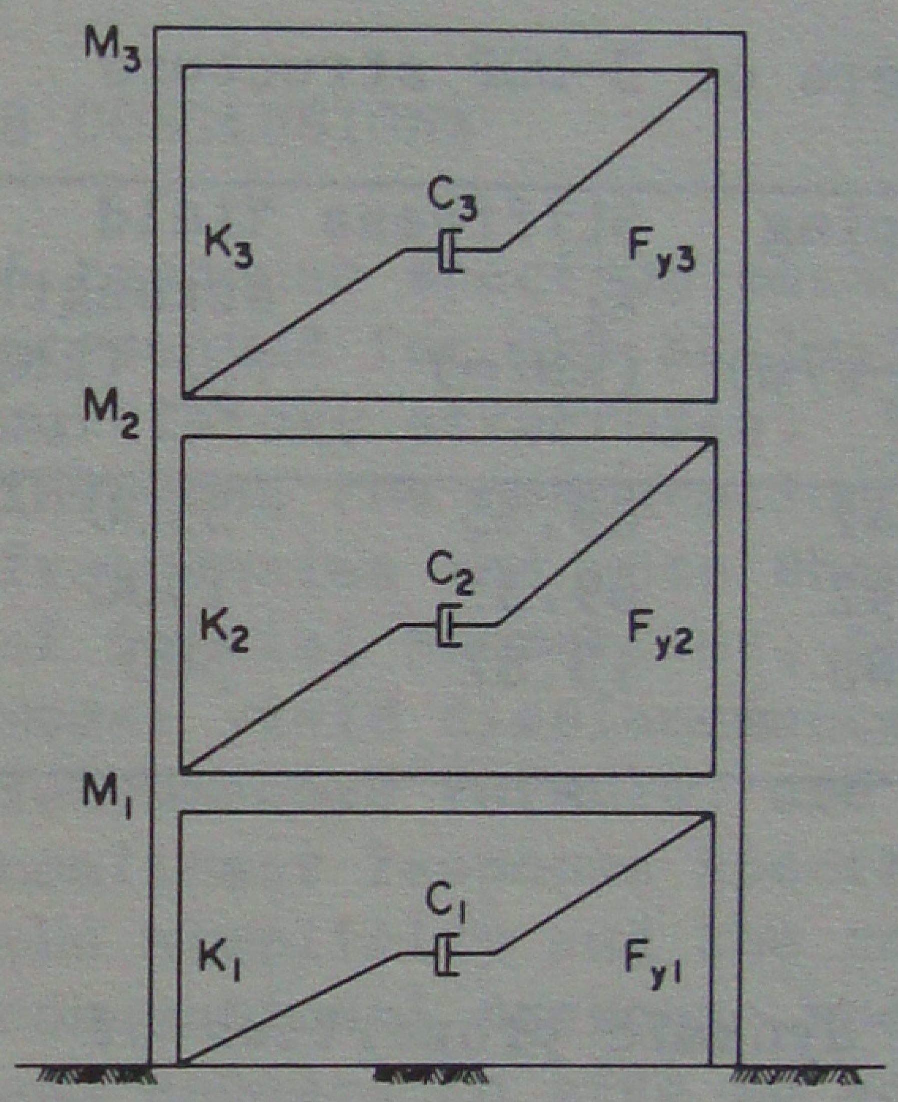

Figure 7. Elastoplastic 3-DOF structure

6 NUMERICAL EXAMPLE

To illustrate the application of the suggested approximate nonlinear modal analysis, consider the shear building shown in Figure 7 with the parameters listed in Table 3, when it is subjected to the first 15.18 sec. of the E-W component of El Centro, 1940, earthquake ground acceleration. Elastoplastic behavior is assumed for each of the floors with the initial stiffnesses and yield strengths indicated in Table 3. The corresponding dynamic properties, on the basis of initial stiffnesses, are given in Table 4.

For this particular building, the established definition of a modal yield displacement, equation 30, gives x_{y1}=0.054m, x_{y2}=0.055m, and x_{y3}=0.070m, with which values and the corresponding natural frequencies and damping ratios one obtains from the response spectra of the earthquake ground motion under consideration (see Figure 1 for the spectrum for 2% damping and Riddell and Newmark (1979) for the spectra corresponding to other values of damping) the following spectral values: SD(1.0, 0.02, 0.054) = 7.3 cm, SD(1.749, 0.035, 0.055) = 5.4 cm, and SD(3.218, 0.064, 0.070) = 0.95 cm. As a consequence, equation 24 gives the following maximum interstory distortions:

$$d_{max} = \{(7.3 \begin{Bmatrix} 0.47689 \\ 0.33146 \\ 0.83556 \end{Bmatrix})^2 + (5.4 \begin{Bmatrix} 0.35883 \\ 0.02388 \\ 1.07127 \end{Bmatrix})^2 + (0.95 \begin{Bmatrix} 0.16440 \\ 0.35537 \\ 0.23570 \end{Bmatrix})^2 \}^{1/2} = \begin{Bmatrix} 3.99 \\ 2.45 \\ 8.41 \end{Bmatrix} cm$$

Table 3. Parameters of 3-DOF structure

Floor	Mass M_i (Mg)	Damping C_i (kN-s/m)	Stiffness K_i (kN/m)	Yield strength F_{yi} (kN)
1	536	0.442	69.35	3500
2	357	0.442	69.35	1500
3	179	0.088	13.87	500

Table 4. Initial dynamic properties of 3-DOF structure

Mode	Natural frequency (Hz)	Gen. mass (Mg)	Damping ratio
1	1.000	838.9	0.020
2	1.749	206.2	0.035
3	3.218	27.86	0.064

	1	2	3
Mode shape	0.47689	0.35883	0.16440
	0.80835	0.38271	-0.19097
	1.64391	-0.68856	0.04473

Note: mode shapes are normalized so that participation factors are unity

7 COMPARATIVE ANALYSIS

To assess the accuracy of the proposed procedure, the maximum floor distortions of a 3 and a 10-DOF structure are calculated with such an approximate procedure when the base of the structures are subjected to three different earthquake ground motions and compared with the corresponding solutions obtained by a more accurate step-by-step numerical integration. The structures are shown in Figures 7 and 8 and their parameters are listed in Tables 3 and 5. Their dynamic properties are respectively summarized in Tables 4 and 6. The ground motions considered are: (a) first 15.18 sec. of the E-W component of El Centro, 1940, earthquake; (b) first 18.20 sec. of the N86E component of Olympia, 1949, earthquake; and (c) first 10 sec. of the N21E component of Taft, 1952, earthquake. In both cases it is assumed that the structures are of the shear-beam type with elastoplastic load-deformation behavior defined by an initial stiffness and a yield strength. All modes are considered for the structure with 3 degrees of freedom and the first five for the one with 10. The numerical integrations are carried out with the computer program DRAIN-2D (Kannan and Powell 1975) using a time step of 0.0025 sec.

Table 5. Parameters of 10-DOF structure

Floor	Mass M_i (Mg)	Damping C_i (kN-s/m)	Stiffness K_i (kN/m)	Yield strength F_{yi} (kN)
1	179	3977	249880	3250
2	170	3773	237050	3000
3	161	3574	224570	2000
4	152	3375	212090	1750
5	143	3177	199620	1500
6	134	2978	187140	1250
7	125	2780	174660	1150
8	116	2581	162190	1100
9	107	2383	149710	1050
10	98	2184	137240	1000

The results of the analysis are presented in Tables 7 and 8 where are also included the average approximate to exact ratios for the three earthquake excitations considered. Since a comparison on the basis of a single ground motion is meaningless and inconsistent with the average response spectra used in design practice, these average ratios are calculated to give a more realistic measure of the effectiveness of the method.

Table 6. Initial dynamic properties of 10-DOF structure in first 5 modes

Mode	Natural frequency (Hz)	Gen. mass (Mg)	Damping ratio
1	1.000	1108.9	0.050
2	2.653	163.0	0.133
3	4.302	57.1	0.215
4	5.868	26.7	0.293
5	7.306	14.1	0.365

Mode shape	1	2	3	4	5
	0.1754	0.1812	0.1671	0.1453	0.1188
	0.3551	0.3342	0.2510	0.1493	0.0550
	0.5342	0.4255	0.2008	-0.0001	-0.1001
	0.7078	0.4324	0.0362	-0.1581	-0.1041
	0.8709	0.3482	-0.1588	-0.1624	0.0586
	1.0185	0.1846	-0.2781	0.0017	0.1378
	1.1458	-0.0301	-0.2500	0.1758	-0.0002
	1.2480	-0.2548	-0.0790	0.1791	-0.1485
	1.3205	-0.4435	0.1511	-0.0059	-0.0667
	1.3589	-0.5532	0.3160	-0.2015	0.1321

Note: mode shapes normalized so that participation factors are unity

Table 7. Approximate and exact maximum story distortions in centimeters of 3DOF structure

Story	Earthquake El Centro App.	Ex.	Olympia App.	Ex.	Taft App.	Ex.	Ave. Ap/Ex ratio
1	3.99	3.33	3.14	2.47	2.56	2.32	1.19
2	2.45	1.87	2.06	1.80	1.75	1.82	1.14
3	8.41	12.3	6.29	7.92	4.73	6.31	0.74

Table 8. Approximate and exact maximum story distortions in centimeters of 10-DOF structure

Story	Earthquake El Centro App.	Ex.	Olympia App.	Ex.	Taft App.	Ex.	Ave. Ap/Ex ratio
1	1.54	1.15	0.92	0.88	0.76	0.78	1.12
2	1.57	1.09	0.93	0.88	0.77	0.81	1.15
3	1.56	1.78	0.90	0.86	0.76	0.81	0.95
4	1.51	1.68	0.87	0.83	0.73	0.79	0.96
5	1.42	1.49	0.83	0.79	0.70	0.75	0.98
6	1.30	1.14	0.78	0.75	0.64	0.85	0.98
7	1.14	0.65	0.72	0.61	0.57	0.67	1.26
8	0.94	0.56	0.63	0.51	0.48	0.57	1.25
9	0.68	0.41	0.49	0.37	0.36	0.43	1.27
10	0.37	0.22	0.28	0.20	0.20	0.23	1.32

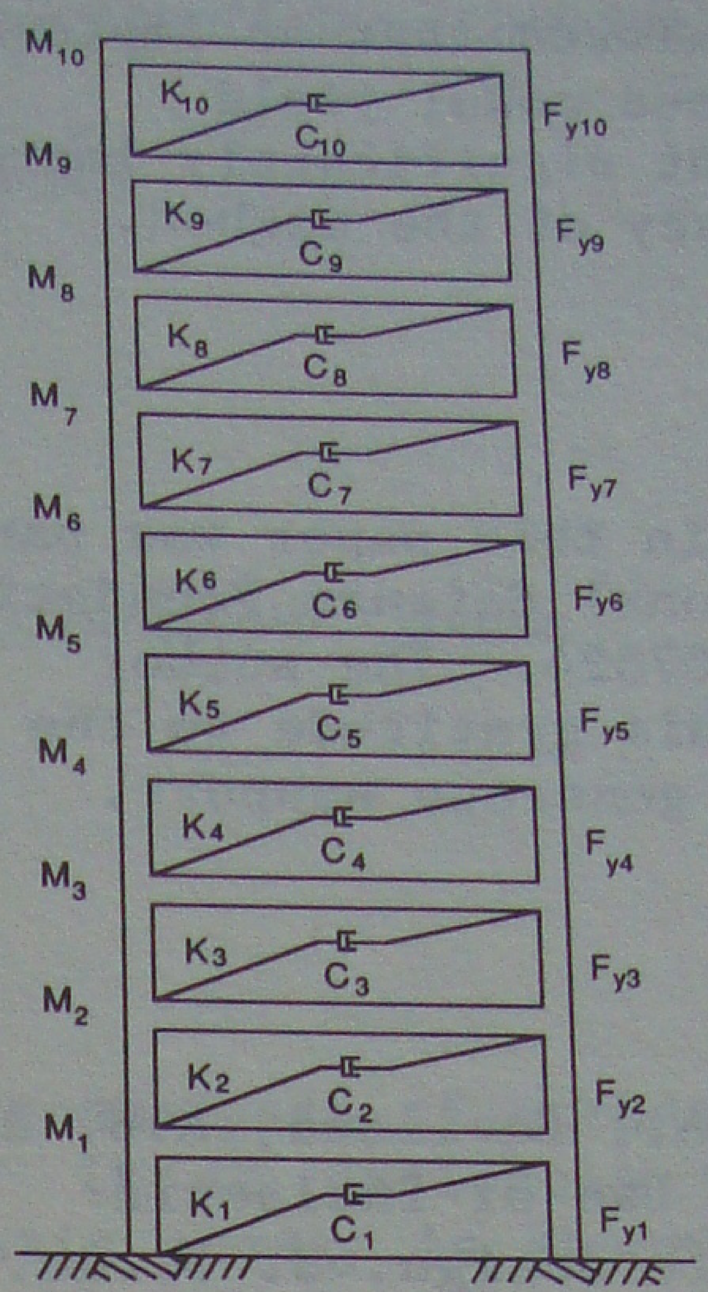

Figure 8. Elastoplastic 10-DOF structure

8 CONCLUSIONS

A response spectrum method has been presented for the analysis of nonlinear multistory structures. The method involves the computation of natural frequencies and mode shapes on the basis of initial properties, the calculation of modal yield displacements according to a recommended formula, and readings from nonlinear response spectra. It maintains the simplicity and the convenience of the conventional response spectrum technique, and appears to give an accuracy that is more or less consistent with the accuracy attained in the analysis of linear structures and certainly adequate enough for preliminary designs. Notwithstanding, only shear systems with elastoplastic load-deformation behavior have been considered, and thus it would be convenient to extended it to include flexible systems with a more sophisticated hysteretic model before it can be put into practical use. Along the same terms, since the performed comparative analysis has been somewhat limited in extent, it is recommendable to examine its accuracy for a wider variety of structures and ground disturbances before any realistic claim on its effectiveness can be made. Lastly, it is desirable to study a little more in depth the influence of the derived definition for modal yield displacements in the accuracy of the proposed method and its adequacy to reflect the true behavior of a nonlinear single-degree-of-freedom system. It is believed that an improved definition for these modal yield displacements might significantly improve the overall accuracy of the method.

AKNOWLEDGEMENTS

The work reported in this paper was partly funded by the National Science Foundation under Grant CEE-8307524. The author wishes to express his gratitude to the Foundation for its generous support.

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